Electric flux and Gauss’s law

The concept of electric flux ($\Phi_E$) is scalar and defined as the dot product of electric field and an area vector.

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

The area vector is simply a vector which points perpendicular to the area it represents and has a magnitude of the area itself.

So let’s say we have a uniform electric field of 10N/C pointed rightwards and passing through an area of 2m$^2$ as shown below.

You can see that the electric field and area vectors are parallel, so the electric flux is:

$$\Phi_E = \mathbf{E} \cdot \mathbf{A} = |E| \cdot |A| \cdot \cos \theta = (10 \text{N/C})(2 \text{m}^2)(\cos 0^\circ) = 20 \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

Answer Webassign Question 1
This idea of electric flux is important because it is an integral part of one of Maxwell’s four fundamental equations of electromagnetism – one known as Gauss’s law.

Gauss’s law is often written as:

\[ Q_{\text{enclosed}} = \varepsilon_0 \cdot \Phi_E \]

Read as, “The charge enclosed by a surface equals the permittivity of free space \((\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N\cdot m^2})\) times the electric flux which passes through that surface.

It can also be written in integral form:

\[ Q_{\text{enclosed}} = \varepsilon_0 \oint E \cdot dA \]

Where the double integral with a loop simply indicates an integral taken over a closed surface.
Let’s take an easy example and enclose a single point charge, \(+Q\), with a radially symmetric, spherical Gaussian surface of radius \(R\).

![Gaussian surface diagram]

We know that the electric field lines will point radially outward from the charge and we can draw all area vectors pointing outward from the Gaussian surface.

![Electric field and area vectors diagram]

This makes \(\mathbf{E} \cdot \mathbf{dA} = |\mathbf{E}| |\mathbf{dA}| \cos 0^\circ = \mathbf{E} \cdot \mathbf{dA}\)

The surface area of a sphere is \(4\pi R^2\), so if we integrate this flux over the entire area, we get:

\[
\oint E \cdot dA = E \cdot 4\pi R^2
\]

So, by Gauss’s law:

\[
Q_{\text{enclosed}} = +Q = \varepsilon_0 \cdot E \cdot 4\pi R^2
\]

which becomes

\[
E = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{R^2}
\]

Answer Webassign Question 4

Answer Webassign Question 5
This is the same as what we had before for the electric field of a point charge:

\[ E = \frac{k \cdot Q}{R^2} \]

where \( k = \frac{1}{4\pi \varepsilon_0} \)

You can see this is true if you put in the values for \( k \) and for \( \varepsilon_0 \).

Look at the equation once more: \( Q_{\text{enclosed}} = \varepsilon_0 \cdot \Phi_E \)

In our example, we had a positive charge enclosed, producing a positive flux (one passing from inside the surface to outside the surface). If we had a lone negative charge in the center of the sphere, it would work just as well. The left hand side would be negative and the right hand side would be negative because the flux would be opposite (passing from outside the surface to inside the surface). You can think of positive flux like a water faucet and negative flux like a water drain.