

1.

a. The block of ice reaches the greater height. Both the block and hoop begin with the same gravitational potential energy, but, for the hoop, some of this energy is translated into rotational kinetic energy rather than linear kinetic energy. So the block reaches the end of the ramp with greater linear kinetic energy, thus a greater linear velocity. Given the same height and launch angle, a projectile with a greater linear velocity will reach a greater height.

b. The block and hoop would reach the same height. Again, both begin with the same gravitational potential energy. Because the hoop does not roll down this time, but rather slides without rolling, none of that potential energy is translated into rotational kinetic energy. Both the block and hoop reach the end of the ramp with the same linear kinetic energy, thus the same velocity. They will then reach the same height as projectiles.

c. Use conservation of energy to determine the velocity at which the block and hoop leave the ramp: $mgh = \frac{1}{2}mv^2$. Then find the x and y components of velocity at which they leave the ramp: $v_x = v \cdot \cos(30^\circ)$ and $v_y = v \cdot \sin(30^\circ)$. The time they will be in flight is v_y/g and the height they reach will be this time multiplied by the average velocity of the ascent: $v_y/2$.

2.

a. $K = K_L + K_R$

$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \left[\frac{2}{5}mR^2 \right] \left[\frac{v^2}{R^2} \right]$$

$$= \frac{1}{2}mv^2 + \left[\frac{1}{5}mv^2 \right]$$

$$= \frac{7}{10}mv^2$$

$$= 1750\text{J}$$

b.i. $1750 = mgh + \frac{7}{10}mv^2$

$$1750 = (25)(10)(3) + \frac{7}{10}(25)v^2$$

$$v = 7.56\text{m/s}$$

b.ii. 25°

c.

$\Delta s = \underline{7.95\text{m}}$	$\Delta s = -3$	$v_{\text{avg}} = -2.6$
$v = 6.85$	$v_i = 3.19$	$a = -10$
$\Delta t = 1.16$	$v_f = -8.38$	$\Delta t = 1.16$

d. The speed would be less than the speed calculated in (b). Friction allows some of the rotational kinetic energy the sphere has along the ground to decrease as it rolls up the incline. Because the change in gravitational potential energy is the same in the ascent, this loss of rotational kinetic energy is transferred into a relative gain of linear kinetic energy in the system with friction.

3.

a. $U_G = K_L + K_R$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \left[\frac{2}{5}mR^2 \right] \left[\frac{v^2}{R^2} \right]$$

$$mgh = \frac{1}{2}mv^2 + \left[\frac{1}{5}mv^2 \right]$$

$$mgh = \frac{7}{10}mv^2$$

$$v = \sqrt{\frac{10gh}{7}}$$

a.i. $K_L = \frac{1}{2}mv^2 = \frac{1}{2}m \left[\frac{10gh}{7} \right] = \frac{5mgh}{7}$

a.ii. $K_R = mgh - \frac{5mgh}{7} = \frac{2mgh}{7}$

b.i. $v_f^2 = v_i^2 + 2(a)(\Delta s)$

$$\frac{10gh}{7} = 0^2 + 2(a) \left[\frac{h}{\sin\theta} \right]$$

$$a = \frac{5g\sin\theta}{7}$$

b.ii. $\Sigma F = ma$

$$F_G + F_{fr} = ma$$

$$mg\sin\theta + F_{fr} = m \left[\frac{5g\sin\theta}{7} \right]$$

$$F_{fr} = \frac{-2mg\sin\theta}{7}$$

c. Still mgh

d. Because the hollow sphere has a greater rotational inertia, more of the gravitational potential energy (mgh) is translated into rotational kinetic energy at a given velocity. This leaves less energy for linear kinetic energy, meaning a lesser linear velocity. At this lower linear velocity, it takes more time to roll down the incline.

4.

a. $L = I\omega = [(2m)(l)^2 + (m)(2l)^2] = 6ml^2\omega$

b. $\tau = rF + rF = -(l)(\mu \cdot 2mg) + -(2l)(\mu \cdot mg) = -4\mu mgl$

c. $\Delta t = \Delta L / \tau = -6ml^2\omega / -4\mu mgl = 3l\omega / 2\mu g$

5.

a. $L_i = L_f$

$$\frac{1}{2}(20M)(4R)^2 \cdot \omega_0 = [\frac{1}{2}(20M)(4R)^2 + (2M)(4R)^2] \cdot \omega$$

$$\omega = \frac{5}{6}\omega_0$$

b. $v = \omega \cdot R = \frac{5}{6}\omega_0 \cdot (4R) = \frac{10}{3}\omega_0 \cdot R$

c. $K = \frac{1}{2}[\frac{1}{2}(20M)(4R)^2 + (2M)(4R)^2] \cdot [\frac{5}{6}\omega_0]^2$

$$K = \frac{200}{3}MR^2 \cdot \omega_0^2$$

d.

	ω increases	ω decreases	ω remains the same
Increase m_{disk}	x		
Increase m_{acrobat}		x	
Increase h			x
Decrease R_{landing}	x		
Land more softly			x

6.

1

$$\text{a. } I_{\text{system}} = 4 \cdot (m)(L^2) + 2 \cdot \frac{1}{12}(2m)(2L)^2 = \frac{16}{3}mL^2$$

$$L_i = L_f$$

$$mvL = \left[\frac{16}{3}mL^2 + mL^2 \right] \omega$$

$$\omega = \frac{3v}{19L}$$

$$\text{b. } v = \omega R = \frac{3v}{19L} \cdot L = \frac{3v}{19}$$

$$\text{c. } K = \frac{1}{2} \left[\frac{16}{3}mL^2 + mL^2 \right] \cdot \left[\frac{3v}{19L} \right]^2 = \frac{3}{38}mv^2$$